Organization

- Time & Date

  Lecture: Mondays 9:15 - 10:45

  Exerc.: every 2nd Monday 15:15 - 16:45

  ⇝ doodle ?

- Lecture
  - via zoom, not recorded
  - I'll upload my notes
  - take your own notes!

- Exercises
  - homework problems every 2 weeks
  - hand in your solutions (properly scanned ...)

- in exercise sessions we discuss solutions and/or I'll upload solutions to moodle and we use the session for questions

- Office hours
  - via email / zoom
  - use moodle forum

- Exam
  - hopefully "live" oral exams
  - alternative: <u>short</u> thesis "Hausarbeit"

- Literature & References
  - in moodle (password "Arnie2k")
  - Arnold, Gusein-Zade, Varchenko
    Singularities of differentiable maps Vol. 1 (& 2)

Plan of the lecture

0. Introduction (today)

1. Background material (AG, AT, BA, CA, DG)

2. Basic notions (critical points, stability, germs of functions etc ...)

3. Classification of singularities

4. Applications (?)
   - Morse theory, Picard-Lefschetz theory
   - Dynamical systems
   - stratified spaces, intersection homology

Consider the two functions $f, g: \mathbb{R} \to \mathbb{R}$

$f(x) = x^2$, $g(x) = x^3$.

Both have a critical point at $x_0 = 0$.

What's the difference?

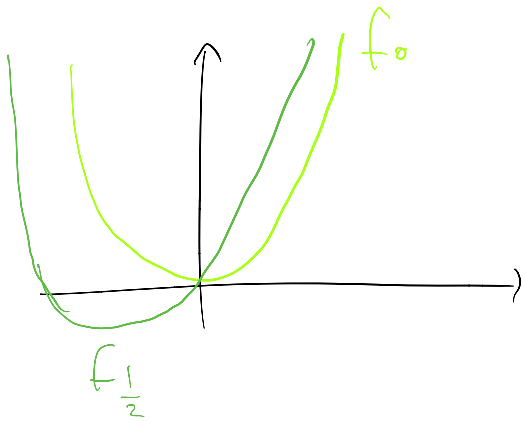
- global behaviour
- local behaviour, minimum vs saddle point

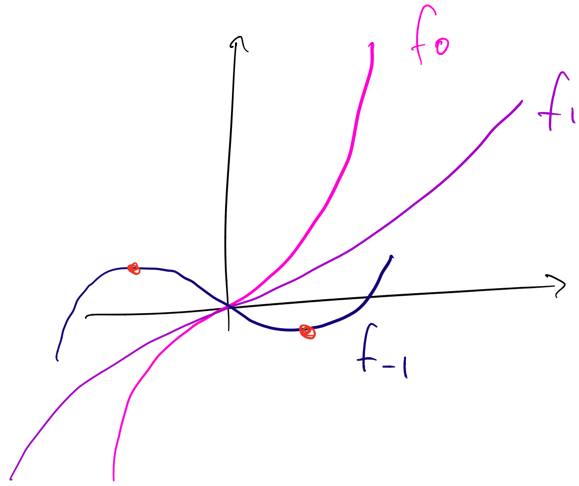
We call a critical point x_0 of $\varphi: \mathbb{R} \to \mathbb{R}$ nondegenerate if $\varphi''(x_0) \neq 0$. otherwise it is degenerate.

These notions are related to the concept of stability:

Let's consider "small" perturbations of f and g:

$$f_\varepsilon(x) = x^2 + \varepsilon x \qquad\qquad g_\varepsilon(x) = x^3 + \varepsilon x$$





$$f_\varepsilon'(x) = 2x + \varepsilon$$

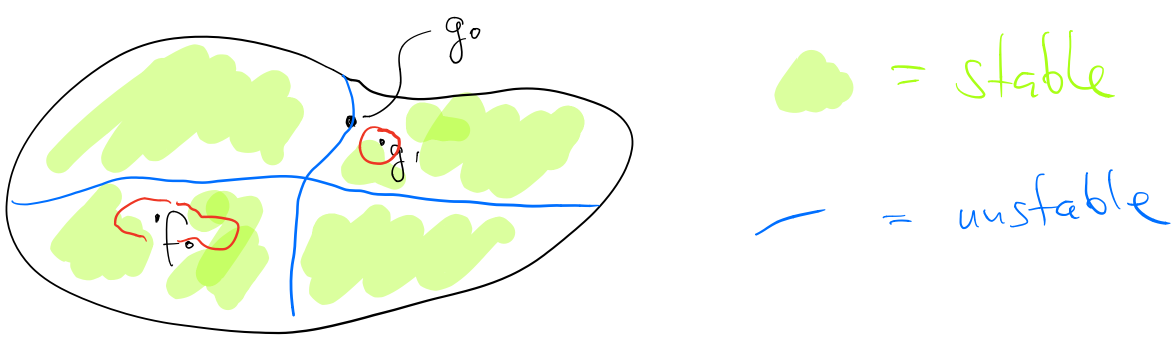
$$\mathrm{Crit}(f_\varepsilon) = \left\{ -\frac{\varepsilon}{2} \right\}$$

$$g_\varepsilon'(x) = 3x^2 + \varepsilon$$

$$\mathrm{Crit}(g_\varepsilon) = \begin{cases} 0 & \varepsilon = 0 \\ \pm\sqrt{\frac{|\varepsilon|}{3}} & \varepsilon < 0 \\ - & \varepsilon > 0 \end{cases}$$

$\Rightarrow$ The critical points of all f_ε and g_ε for $\varepsilon \neq 0$ all stable, while the one for g_0 is <u>not</u>.

The "space of functions on $\mathbb{R}$" "looks like"

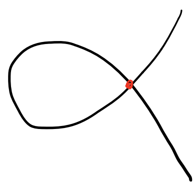


Our (first) goal is to make this statement precise, to describe the different regions in the picture (from an algebraic and topological perspective) and to generalize this to the case of $C^\infty(M,N)$ where M,N are smooth manifolds.

Obviously, the latter depends crucially on the relation between $\dim M$ and $\dim N$ ...

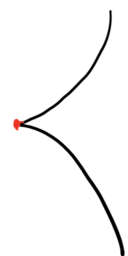
e.g.

"double point"



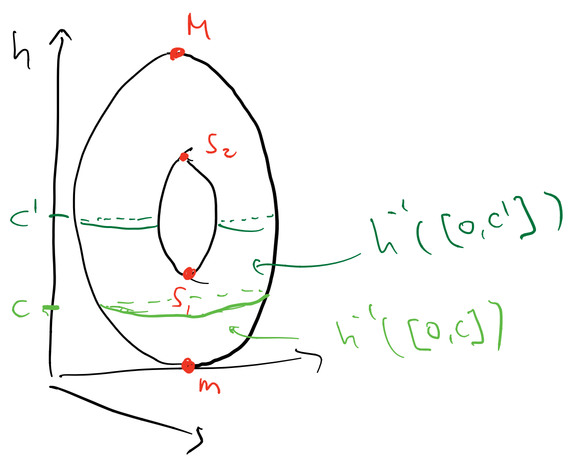
$y^2 = x^2 + x^3$

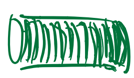
"cusp"



$y^2 = x^3$

h = height function on T^2



$h^{-1}([0,c']) \cong$ 

$h^{-1}([0,c]) \cong$